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ILIJA LUKAČEVIĆ AND HIS DIGITIZED WORKS

Abstract. Ilija Lukačević (1935-2020) was a professor at Faculty of Sciences and Mathematics, at the Department of Mechanics and Astronomy. He was author of several papers, mostly in the field of theory of relativity. Some of these papers and his doctoral dissertation can be found in the Virtual Library of the Faculty of Mathematics in Belgrade, http://elibrary.matf.bg.ac.rs. In this paper we present these works, as well as some details from life of Ilija Lukačević.

Keywords. Ilija Lukačević, digitization, mathematical works.

1. Introduction

Professor Ilija Lukačević was a respectable theoretical mechanic and retired professor of the Faculty of Mathematics in Belgrade. He was known as the expert in special and general theory of relativity, as well as tensor calculus and differential geometry. He published 29 scientific papers and a university textbook *Fundamentals of Relativity* (Osnove teorije relativnosti). He participated in various domestic and international conferencies and he gave several lectures by invitation. We present here a part of his life and work.

2. Biography

Professor Ilija Lukačević was born on November 13, 1935, in Belgrade, where he finished his primary and secondary education. In 1954. he enrolled in undergraduate studies at Department of Mathematics, Mechanics and Astronomy at the Faculty of Natural Sciences in Belgrade (later known as the Faculty of Mathematics). For the first two years he studied astronomy, after which he moved on to mechanics. He graduated in the field of mechanical sciences on February, 1960, afterwards he served in the Army. In 1961, after military service, he enrolled in master studies in mechanics at the same faculty and became a Teaching Assistant at the Department of Mechanics and Astronomy. As a Teaching Assistant, he taught courses in Statics, Oscillation Theory, Analytical Mechanics and Tensor Calculus. In December 1965 he defended his master‘s thesis entitled *On the perfect charged fluid in the general theory of relativity* (O savršenom naelektrisanom fluidu u opštoj teoriji relativnosti). As a French scholarship holder, in the school year 1966/67 Professor Lukačević specialized in the field of relativity mechanics at the Collége de France in Paris, under the supervision of one of the experts in the field of theory of relativity, Professor André Lichnerowicz. In June 1968, under the supervision of Academician Tatomir Anđelić, Professor Lukačević defended his doctoral thesis entitled *Alfvén waves in relativistic magnetohydrodynamics* (Alfenovi talasi u relativističkoj magnetohidrodinamici). The dissertation defence committee was

composed of Academician Tatomir Anđelić (Full Professor), Marko Leko (Assistant Professor) and Đorđe Mušicki (Associate Professor).

Figure 1. Professor Ilija Lukačević.

Two years later, he was elected as Assistant Professor, until 1979 when he was promoted in an Associate Professor. In 1986 he became Full Professor. Until the retirement on April 30, 2001, Professor Ilija Lukačević held lectures in Statics, Elements of Rational Mechanics, Theory of Relativity, Magnetohydrodynamics, Rational Mechanics, Analytical Mechanics, Tensor Calculus with Applications in Mechanics and Celestial Mechanics and Theory of Movement of Earth‘s Artificial satellites.

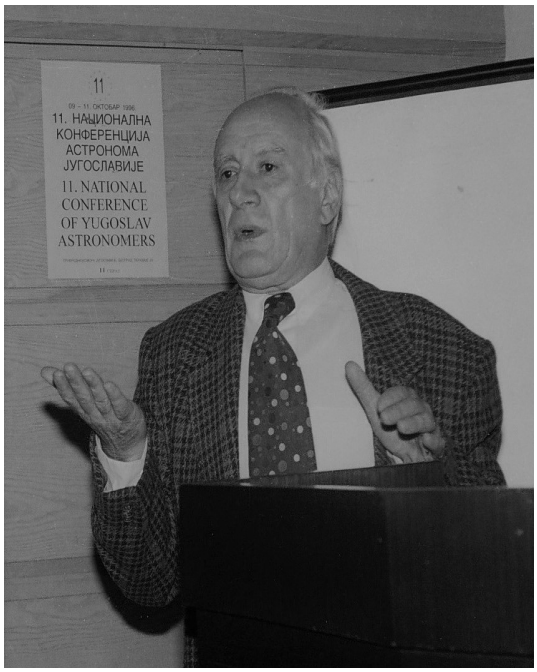
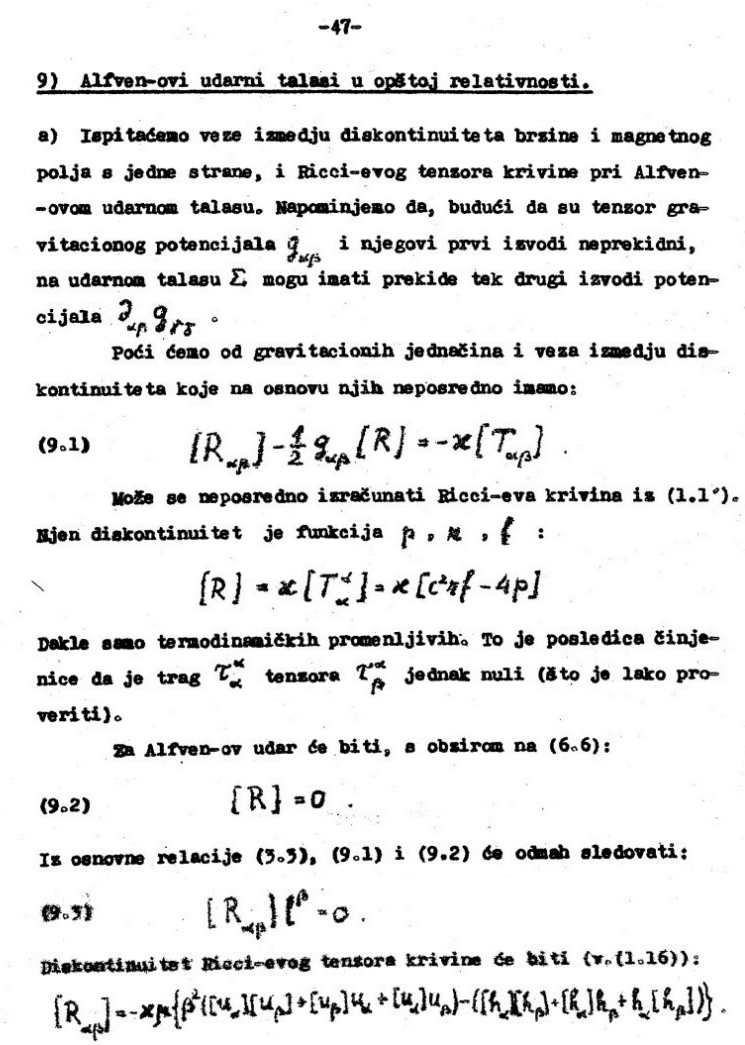


Figure 2. Professor Lukačević as a lecturer on the XI National Conference of Yugoslav Astronomers, 9 - 11.10.1996.

Professor Ilija Lukačević published 29 scientific papers, mostly on the magnetohydrodynamics and theory of relativity. He also published the textbook *Fundamentals of Relativity*. He was a participant in 14 national and 6 international congresses

and conferences. Professor Lukačević gave lectures by invitation once in Budapest (May 1980) and Paris (1982), as well as twice in Moscow (in autumn 1974 and 1981).

He was a member of the International Astronomical Union, Yugoslav Society for Mechanics and Society of Astronomers of Serbia. Moreover, he was the director of the Institute of Mechanics. Also, he wrote reviews for Mathematical Reviews, as well as for Serbian Astronomical Journal.



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9) Alfven-ovi udarni talasi u opštoj relativnosti.

a) Ispitaćemo veze izmedju diskontinuiteta brzine i magnetnog polja s jedne strane, i Ricci-evog tenzora krivine pri Alfven-ovom udarnom talasu. Napominjemo da, budući da su tenzor gravitacionog potencijala $g_{\alpha\beta}$ i njegovi prvi izvodi neprekidni, na udarnom talasu Σ mogu imati prekide tek drugi izvodi potencijala $\partial_{\alpha\beta} g_{\gamma\delta}$.

Poći ćemo od gravitacionih jednačina i veza izmedju diskontinuiteta koje na osnovu njih neposredno imamo:

(9.1) $$[R_{\alpha\beta}] - \frac{1}{2} g_{\alpha\beta} [R] = -\varkappa [T_{\alpha\beta}] .$$

Može se neposredno izračunati Ricci-eva krivina iz (1.1'). Njen diskontinuitet je funkcija p, $\varkappa$, ℓ :

$$[R] = \varkappa [T^{\alpha}_{\alpha}] = \varkappa [c^2 \eta \ell - 4p]$$

Dakle samo termodinamičkih promenljivih. To je posledica činjenice da je trag τ^{α}_{α} tenzora τ^{α}_{β} jednak nuli (što je lako proveriti).

Za Alfven-ov udar će biti, s obzirom na (6.6):

(9.2) $$[R] = 0 .$$

Iz osnovne relacije (3.3), (9.1) i (9.2) će odmah slediti:

(9.3) $$[R_{\alpha\beta}] \ell^{\beta} = 0 .$$

Diskontinuitet Ricci-evog tenzora krivine će biti (v. (1.16)):

$$[R_{\alpha\beta}] = -\varkappa \mu \left\{ \beta^2 ([u_{\alpha}][u_{\beta}] + [u_{\beta}] u_{\alpha} + [u_{\alpha}] u_{\beta}) - ([h_{\alpha}][h_{\beta}] + [h_{\alpha}] h_{\beta} + h_{\alpha} [h_{\beta}]) \right\} .$$

Figure 3. Page from the doctoral thesis Alfvén waves in relativistic magnetohydrodynamics.

Professor Ilija Lukačević was a mentor to Dragi Radojević, scientific associate of the Mathematical Institute, whose doctoral dissertation was in the field of general relativity. Professor Lukačević was a member of the comittee for the defence of the theses of the Department of Astronomy, namely the PhD thesis of Trajko Angelov in 1981 and the MSc thesis of Zlatko Ćatović in 1990 (as a mentor). He was a regular participant of the seminar of the Department of Astronomy, as well as joint meetings of the Department of Astronomy and the Department of Mechanics on Mondays. In the middle of 1980s, Professor Lukačević's interest in alternative theories of gravity began, especially in Rosen's bimetric theory of gravity, where he gave the main results and contributions. At that time, he published several papers on that topis in the respected international journal Gen. Relativ. Gravit., one of them was co-authored with Zlatko Ćatović.

Professor Ilija Lukačević was honest and modest man, who aproached science universally and with original ideas and methods. He passed away on June 14 2020 in Belgrade. Colleagues and students will remember him with love, respect and appreciation.

3. Textbook *Fundamentals of Relativity*

This book is primarily a textbook for the undergraduate course that was taught by Professor Lukačević at the Department of Mathematical, Mechanical and Astronomical Sciences of the Faculty of Science and Mathematics in Belgrade, and after the Department grew into the Faculty of Mathematics in the early 1990s as well.

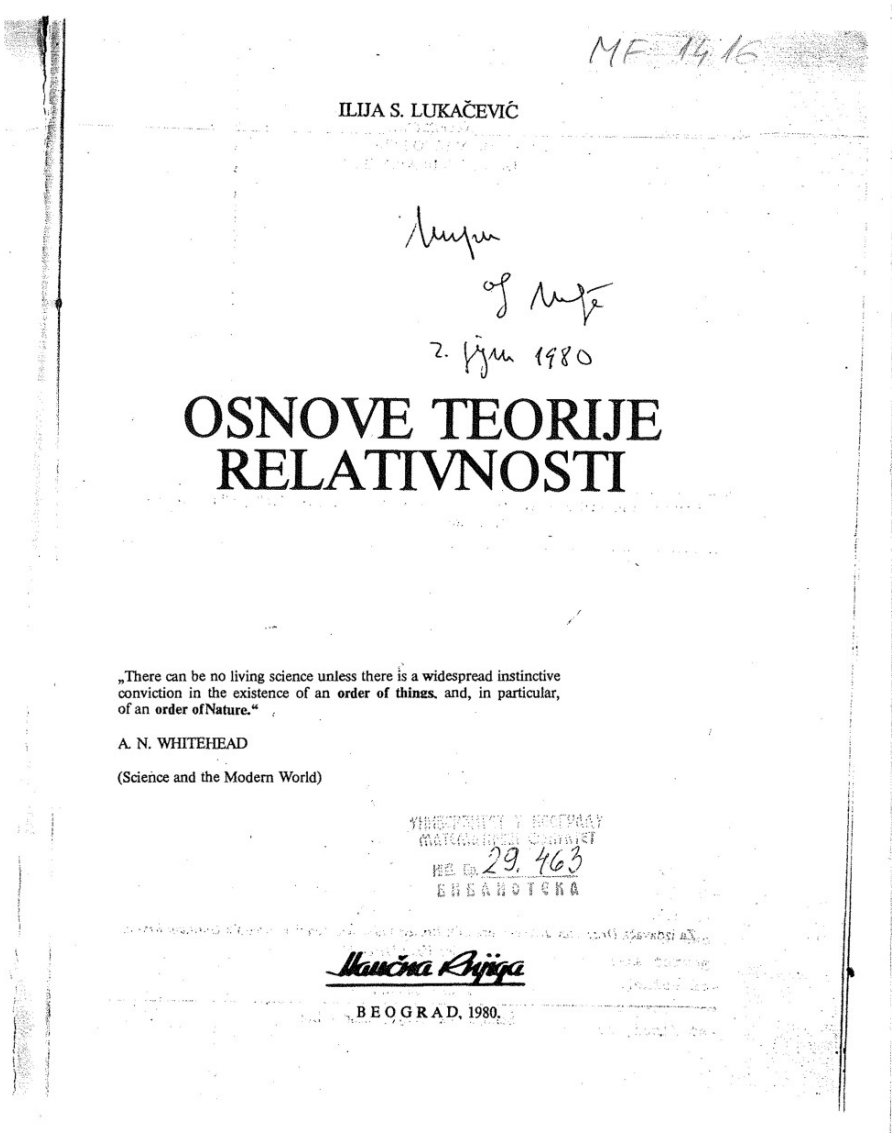
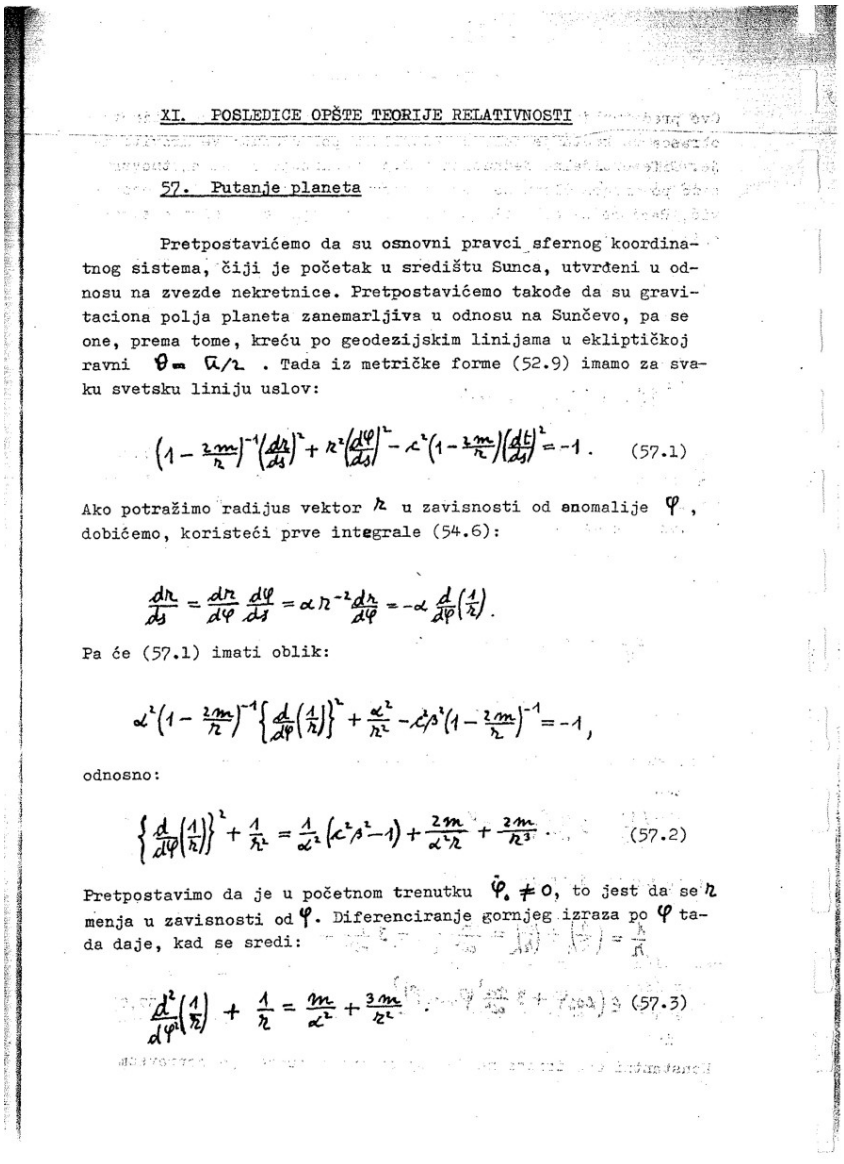


Figure 4. The cover page of the university textbook Fundamentals of Relativity.

The course was obligatory for theoretical mechanics students and elective for students of astronomy and mathematics. However, the author wrote the textbook with greater ambitions, so some parts of the textbook were not included into the regular program of undergraduate studies. Therefore, the textbook has a monographic character.

In teaching the theory of relativity, there are mainly two approaches. The first approach is of an empirical character, and in that way the theory of relativity is presented as an additional topic to the courses of general and theoretical physics. In that case, this theory is mostly presented as a modified Newtonian physics. The second approach is of a deductive character, in which the theory of relativity is presented as a mathematical theory based on several postulates, that is, as a part of differential geometry, only with incidental remarks about physics. Today, this second approach is mostly present and is particularly represented in scientific and monographic literature. Although, according to Professor Lukačević, this textbook does not belong to either of those two approaches, nevertheless, with a cursory insight, the reader is quickly convinced that the second methodology is mostly represented in the textbook. In support of that is also the mathematical tools used by the author, primarily the methods of differential geometry and tensor calculus.



XI. POSLEDICE OPŠTE TEORIJE RELATIVNOSTI

57. Putanje planeta

Pretpostavićemo da su osnovni pravci sfernog koordinatnog sistema, čiji je početak u središtu Sunca, utvrđeni u odnosu na zvezde nekretnice. Pretpostavićemo takođe da su gravitaciona polja planeta zanemarljiva u odnosu na Sunčevo, pa se one, prema tome, kreću po geodezijskim linijama u ekliptičkoj ravni $\vartheta = \pi/2$. Tada iz metričke forme (52.9) imamo za svaku svetsku liniju uslov:

$$\left(1-\frac{2m}{r}\right)^{-1}\left(\frac{dr}{ds}\right)^2 + r^2\left(\frac{d\varphi}{ds}\right)^2 - c^2\left(1-\frac{2m}{r}\right)\left(\frac{dt}{ds}\right)^2 = -1 . \qquad (57.1)$$

Ako potražimo radijus vektor r u zavisnosti od anomalije φ, dobićemo, koristeći prve integrale (54.6):

$$\frac{dr}{ds} = \frac{dr}{d\varphi}\frac{d\varphi}{ds} = \alpha r^{-2}\frac{dr}{d\varphi} = -\alpha\frac{d}{d\varphi}\left(\frac{1}{r}\right).$$

Pa će (57.1) imati oblik:

$$\alpha^2\left(1-\frac{2m}{r}\right)^{-1}\left\{\frac{d}{d\varphi}\left(\frac{1}{r}\right)\right\}^2 + \frac{\alpha^2}{r^2} - c^2\beta^2\left(1-\frac{2m}{r}\right)^{-1} = -1,$$

odnosno:

$$\left\{\frac{d}{d\varphi}\left(\frac{1}{r}\right)\right\}^2 + \frac{1}{r^2} = \frac{1}{\alpha^2}\left(c^2\beta^2-1\right) + \frac{2m}{\alpha^2 r} + \frac{2m}{r^3} . \qquad (57.2)$$

Pretpostavimo da je u početnom trenutku $\dot{\varphi}_0 \neq 0$, to jest da se r menja u zavisnosti od φ. Diferenciranje gornjeg izraza po φ tada daje, kad se sredi:

$$\frac{d^2}{d\varphi^2}\left(\frac{1}{r}\right) + \frac{1}{r} = \frac{m}{\alpha^2} + \frac{3m}{r^2} . \qquad (57.3)$$

Figure 5. Page from the textbook Fundamentals of Relativity.

The book consists of two parts, the special theory of relativity and the general theory of relativity. In the first part, the standard foundations of the special theory of relativity are presented, starting with Lorentz transformations and ending with the theory of linear operators in Minkowski space – the mathematical model of this theory.

Of particular interest and value in this chapter is the continuum mechanics theory, which is not presented in basic textbooks on the theory of relativity. Electromagnetic theory is also presented in this framework, in which Maxwell's equations play a central role. Experiments that confirm the validity of this theory are also mentioned.

The second part is devoted to the general theory of relativity and some of its applications, primarily in cosmology. This theory is also known as the theory of gravity, and hence the first and main place is given to the gravitational field equations, also known as Einstein's field equations. At the end of this book, Hilbert's derivation of these equations using the calculus of variations is given, which places this theory on a strict mathematical foundations. In this part, reader can find explanations of many phenomena related to gravity, such as gravitational waves and black holes, which are today central topics in the research of the universe and about which we find popular articles and reports even in the daily press and media. It should be mentioned here that most of these physical phenomena were predicted using mathematical apparatus within the framework of the general theory of relativity long before they were experimentally proven. For example, gravitational waves were predicted by Oliver Heaviside at the end of the 19^{th} century, then by Poincaré in 1905 and by Einstein based on the theory of relativity in 1916. The first direct evidence that this phenomenon really exists was measured in 2015 as a result of the merger of two black holes in a distant part of the universe. The value of this textbook follows from the fact that these elements of the special and general theory of relativity, which are today the focus of the most advanced mathematical research, are presented in details. When it comes to applications in cosmology, the theory that is today known as the standard model (ΛCDM model) with the cosmological constant Λ (the so called Einstein's Λ constant) is presented in the textbook, as well as basic solutions, such as Einstein-de Sitter's and de-Sitter's models.

Although the author optimistically stated in the introduction that the textbook can be read (at least in parts) by secondary school teachers and students of physically and technically oriented sciences, in order to do so, the reader must have strong base in theory of differential geometry and tensor calculus. Considering the brevity and rigour of the presented topics, it is necessary for the reader to be patient and persistent. Consequently, the reader will be rewarded with an essential understanding of perhaps today's most brilliant theory and the most profound product of the human spirit. These words describe in a great amount the value of the textbook and also erudition of the author, Professor Lukačević. Primarily due to the difficulty of the presented topics, the textbook probably did not have a great impact on our professional public. There are exceptions, of course. In the middle of 1980s, a one-year seminar was organized, consisting of three members, Professor Lukačević, Dragi Radojević and one of the authors of this article, Žarko Mijajlović.

The textbook has 258 pages, a large number of references, some of which were scientific works at the time of writing the textbook. It also incudes selected tasks that illustrate and complement the main content in the textbook. The textbook was printed in 1980. At the end of the textbook, the author thanks his colleague from the Department of Mechanics, Professor Marko Leko, and his professor, Academician Tatomir Anđelić, who read the textbook and at the same time were a reviewers.

The textbook has been scanned and a digital copy has been placed in the Virtual Library of the Faculty of Mathematics. We note that in the Virtual Library can also be found the doctoral dissertation *Alfvén waves in relativistic magnetohydrodynamics*

4. Seminar for the theory of relativity and cosmological models

Professor Lukačević, together with Žarko Mijajlović and Dragi Radojević, founded a seminar for the theory of relativity and cosmological models in 2014. The seminar was held at the Faculty of Mathematics until 2017, when it was transferred to the Mathematics Institute of the Serbian Academy of Sciences. Professor Lukačević regularly participated in this seminar, both as a listener and a lecturer. Professor Ilija Lukačević, even in his late retirement days, still gave dynamic lectures with a lot of formulas that he wrote down without reminders. The main theme of his lectures was the general theory of relativity. Brisk discussions on various topics that were discussed at the seminar, continued after the end of the seminar meeting, usually in a neaby cafe. On those occasions, Professor Lukačević's oppinions were listened carefully and with respect.

Figure 6. Prof. Lukačević with the photograph of his ancestor, Colonel Vojislav Anđelković.

On one such occasion, Professor Lukačević spoke with love and pride about his ancestor, Colonel Vojislav Anđelković, a participant in the Balkan Wars and the First World War. Colonel Anđelković was awarded with numerous domestic and foreign medals, which Professor Ilija Lukačević and his brother Stefan Lukačević had donated to the Office for Medals of the Archive of the Serbian Academy of Sciences and Arts, in October 2019. The last Professor Lukačević's lecture on the seminar was held on May 29, 2019, entitled *On the visibility of length changes and the measurability of time in relativity* (O vidljivosti promene dužine i merljivosti vremena u relativnosti).

5. Conclusion

In this paper we presented life and work of Professor Ilija Lukačević. Professor Lukačević contributed both to teaching and to scientific segments of the theory of relativity. He improved the teaching of mechanics at the Faculty of Mathematics in Belgrade, while at the same time he had significant results in magnetohydrodynamics and the theory of relativity. He was also interested in the applications of the theory of relativity, especially in cosmology. That interest in cosmology resulted in co-founding of the seminar for the theory of relativity and cosmological models of the Mathematical Institute of the Serbian Academy od Sciences and Arts.

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