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ACADEMICIAN BOGDAN GAVRILOVIĆ AND HIS DIGITIZED WORKS

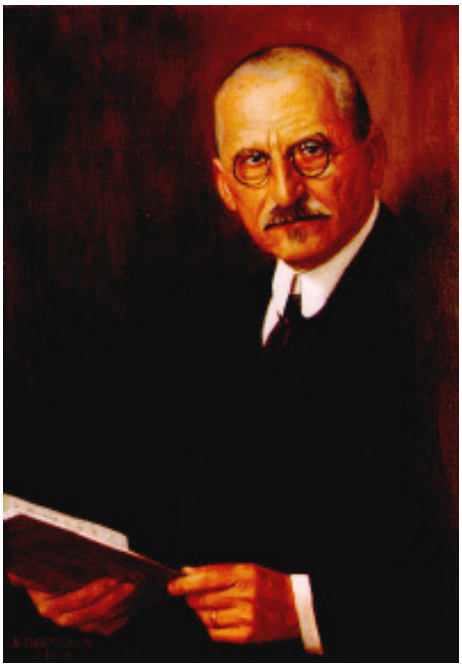
Abstract. Bogdan Gavrilovie was born in Novi Sad on January 1,1864. Top of the class in his generation, after completing secondary education he was sent by *Tekelianum* (a Serbian educational institution) to study at the Philosophical Faculty of the University in Budapest, where he obtained doctor's degree in mathematics in 1887. In the same year he was appointed professor at the High School in Belgrade which in 1905 was promoted to the University of Belgrade. He lived in Belgrade until his death in 1947, active as university professor until 1941. He was elected member of the Serbian Academy of Sciences in 1901 and of the Yugoslav Academy of Sciences in 1906. He has great merits for the foundation of Belgrade University and he was elected twice for the rector of the University. On the end of XIX century he had published two voluminous university textbooks which had the character of monographs: *Analytical Geometry* (1896) on 900 pages, and *Theory of Determinants* (1899) on linear algebra. Both works may be considered as capital works in mathematics in Serbia. Academician Radivoj Kašanin thus wrote of the two books: „Both, especially the latter, would do honour to any nation, and many countries, at that time more powerful and luckier than us, could not boast of such works." The aim of this paper is to present the work and life of Bogdan Gavrilović and his digitized books and papers deposited in the Virtual Library of the Faculty of Mathematics in Belgrade.

1. Introduction

On the layout of 19th and 20th century, when the skyline of new century mathematics was established, a new generation appeared in the Serbian Science. The arrival of the new generation was turning point for the development of mathematical science in Serbia. Before its appearance, there was not true science in Serbia, while to the present day the waves and impact of the work of that generation spread. Among the mathematicians of the late 19th and early 20th century, highly prominent place belongs to the professor of the High School and of the Belgrade University, the academician Bogdan Gavrilović. Along with Dimitrije Nesic and Mihailo Petrović, Gavrilović is most responsible for creating a mathematical school in Belgrade and for the creation of the University of Belgrade.

We digitized his books and most of his scientific papers, speeches and other writings. Digitized works of Bogdan Gavrilović are deposited in the Virtual Library of the Faculty of mathematics in Belgrade http://elibrary.matf.bg.ac.rs where they are freely available to the general public. It is interesting to mention that digitization of Gavrilović's books was, as far as it is known to the author of this paper, the first undertaking in digitization of scientific heritage in Serbia. This project took place already in 1994. Even if there are several articles in Serbian on Gavrilović and his work, this paper is probably the first writing on him in English in somewhat generous extent. The paper is largely based on the article *Bogdan Gavrilović*, appearing in *Život i delo sprskih nučnika*, an edition of SASA (Serbian Academy of Science and Arts), see [1]. Here we will also try to present some elements of personal and scientific biography of Professor Bogdan Gavrilović.

2. Biography



Bogdan Gavrilović was born in Novi Sad on January 01, 1864 (December 20, 1863 by the Julian calendar) by the mother Sophia and the father Alexander. According to some data, Gavrilović's family originates from Herzegovina, namely, academic Radivoj Kašanin in the obituary dedicated to Bogdan Gavrilović mentioned that Gavrilović's great grandfather moved from that area. Childhood and youth Bogdan spent in Novi Sad, or "Serbian Athens" as the spiritual and cultural center of Vojvodina then was called. Gifted and valuable, Bogdan Gavrilović was the best student of Novi Sad high school, and even Jovan Jovanović Zmaj wrote about that in his journal *Neven*, which was among the first class educational Serbian institutions. The teaching was better there than in other places, the old classical languages and literature as well as the natural sciences and mathematics. In addition to talents, a contribution to his education was the upbringing which he received in his family: his grandfather was a teacher and manager of primary school and his father was a professor and director of the Serbian high school in Novi Sad. Having such a good preparation, being the cadet of Serbian institution Tekelianum, Gavrilović graduated with ease and great success in the Faculty of Philosophy, University of Budapest, on the Department of Mathematics, Physics and Astronomy.

In his twenty-third year, on the University of Budapest, Gavrilović defended the doctoral thesis on analytic functions "On representation of one-branch analytic functions" in Hungarian ("Az egyértékü analytikus függvények elöállitásairól"), thus received a degree of doctor of mathematical sciences on June 11, 1987. Prior to that, as a condition for obtaining a doctorate, Gavrilović has passed the exam in mathematics (June 1986 before the commission composed of G. Kondor and A. Scholtz), astronomy (June 1887 before G. Kondor) and experimental physics (June 1887 before Eötvös L. and A. Scholtz). Let's mention that in the 19th century there was a total of six Doctors of Mathematics of Serbian origin. Aside from Bogdan Gavrilović those were Dimitrije Danić, Vladimir Varićak, Djordje Petković, Mihailo Petrović and Peter Vukićević, see [2].

After Budapest, Gavrilović improved and stayed in the famous scientific centers in Western Europe: Germany, Switzerland and France. So, in Berlin he attended lectures by the famous mathematician Karl Weierstrass.

In addition to scientific, Gavrilović was the sporting spirit. As a high school student he was the member of the Novi Sad Gymnastics Society which was founded by the poet Laza Kostić. As a student, a young man with a beautiful appearance and high physique, he spend some time in Prague on the famous sports school where he trained in gymnastics and fencing. His interest in sports will continue. So, later in Belgrade he will be the president of the Belgrade Society for gymnastics and fencing, and then, in the last decade of the 20th century, the founder and president of the sports society "Sokol" in Belgrade. It may be that he looked up to his teacher Weierstrass who at one time, early in his career, was a teacher of gymnastics, as well as a good swordsman.

Bogdan Gavrilović, as a young mathematician in the rise, came to Belgrade in 1887 where he spent sixty years of uninterrupted activities until his death in 1947. He immediately became teacher of the High school (in competition with Dimitrije Danić), and in 1892 the honorary (full) professor. At that time the Great School, which under the law of organization was "scientific institution for high and professional education", after the Lyceum reform in 1863 was divided into three faculties: Philosophy, Law and Technical. The school was situated in, for that time a large building which is now known as Captain Miša building. This building has been endowed to the people for the High School purposes by Captain Miša Anastasijević. At a time when Gavrilović began teaching, there were two sections at the Faculty of Philosophy: historical-philosophical and Natural Sciences. It is interesting that the mathematics and natural sciences were taught only at the Technical Faculty since 1873. With introduction of mathematics at the Faculty of Philosophy, the Department of Mathematics has been established. Dimitrije Nešić was the only teacher in this department until the arrival of Bogdan Gavrilović. At that time the subject of mathematics at the Great School was divided into two Departments: the higher and lower mathematics. Department of higher mathematics took Dimitrije Nešić while the Department of lower Mathematics went to Bogdan Gavrilović. Bogdan Gavrilović at the time of his employment did not have the citizenship of the Kingdom of Serbia. Therefore, under the decree of King Milan he was set for "contractual suplent of Faculty of Philosophy of the High School under the foreigner conditions" (Educational Gazette 8 (1887), pp. 682nd). Let's mention that the teachers at that time could have had the title of professor, assistant professor and suplent. There was also the title of honorary professor which the minister himself could set independently.

In his course of mathematics, Gavrilović taught analytic geometry, trigonometry and foundation of the school mathematics. At the time, a textbook in trigonometry in Serbian language already existed. The textbook was written by Dimitrije Nešić and published in 1875. Gavrilović used this book for his trigonometry course, and at the same time, along with lectures, prepared a textbook on analytical geometry, which will prove that it was his most extensive work.

In the Grand School, which became Belgrade University in 1905, Gavrilović held classes for fifty years, along with Mahailo Petrović, a prominent Serbian mathematician. Mihailo Petrović, five years younger than Gavrilović, returned from France to his homeland in 1894 and became the professor of mathematics at the Grand School, immediately after receiving a doctorate at Paris Sorbonne. That same year the teaching of mathematics for the students of the Technical faculty which Bogdan Gavrilović took, has been distinguished. Mihailo Petrović remained on the Faculty of Philosophy.

According to general opinion, which Milutin Milanković also highlights in the obituary dedicated to Bogdan Gavrilović, two of them set the foundation of our mathematical school. Gavrilović and Petrović were complementary in their interest in mathematics. Petrović was primarily a professor of analysis with applications and not much interested in algebra and geometry. In contrast, Bogdan Gavrilović dealt with algebra, geometry and function theory (complex variables). In the algebra area he was particularly interested in combinatorics and number theory. His teaching activities in geometry, which was accessed through the Descartes analytical methods coordinates, presented in his main work, then of the great scientific value, "Analytic Geometry", published in 1896. This classic mathematical discipline was already quite known area of mathematics, and there was not much room for new discoveries, as opposed to areas where Petrović worked. However, Bogdan Gavrilović in his book "Theory of the determinants" published in 1899, as in the twenty papers published in "Glas" of the Serbian Academy of Sciences and "Radovi" of the Academy of Sciences and Arts in

Zagreb, has showed his deep knowledge, a broad scholarship and real mathematical skill.

As time went by, views and interest of Bogdan Gavrilović had spread beyond the mathematical sciences. Mathematician by studies, humanist by education, he knew well two classical languages and the important modern languages. As for the mathematics itself, he was equally interested in how the natural and mathematical sciences reflect the different aspects of the development of human society and civilization, but also the cultural events in the world.

His thoughts on these aspects of the exact sciences, Gavrilović presented in a series of speeches and essays which titles speak for themselves: "Science and Civilization", "Social mission of the University", "Culture and Harmony", "On the eighteenth-century rationalism and its impact on society of the time", "The history as the science and its meaning". However, among these writings a pearl of rhetoric stands aside; that is approaching to academy sermon about the problem of space, hyper-space and continuum, which was held in the Academy on March 7, 1926. It will be thoroughly discussed later. Reading these documents we learned that Bogdan Gavrilović was not only a scientist but also the educator and cultural activist with the European understanding of science and culture. This activity of his greatly contributed to the establishment and development of our scientific institutions and the enlightenment of our people.

Let's highlight briefly what Bogdan Gavrilović achieved in this field.

Already in 1894 the library of the Mathematics seminar has been established that provided generations of mathematicians of the Belgrade University with ample opportunities for scientific work. Until the First World War the library was led by Bogdan Gavrilović and Mihailo Petrović, and later other mathematicians participated. The library disposed of the rich fund of books, sets of journals, monographs and other mathematical literature. When the scientists of the Faculty of Philosophy in 1938 moved to a new building built next to the old building of the Faculty of Philosophy in Captain Misa's building, the library has been moved in the same place. Unfortunately, only two days before the liberation of Belgrade, on October 18, 1944, the retreating enemy army set fire in library, and it was destroyed. All that was left were a few books that were found borrowed by individuals. From the saved first book of inventory up to the 1907, it can be seen that Bogdan Gavrilović manually wrote down the books, up to the number of 110, and Mihailo Petrović over that number up to 301.

The story of the Bogdan Gavrilović life path is inseparable from the development of the Belgrade University and mathematical sciences in Serbia. In the late 19th century and especially in the early 20th century came rapid development of high education in Serbia. So, in 1896 the Great School gained certain autonomy, and professional level teaching of mathematics approached to the European level. In the 1900, legal regulation further enhanced the level of teaching and scientific work. So, faculties become independent units of the Great School and get the organization that will remain until after the Second World War. In 1900, the Seminar for mathematics, mechanics and theoretical physics has been created, in work of which teachers of mathematics, mechanics and astronomy from the Technical Faculty of Philosophy participated. Mihailo Petrović and Bogdan Gavrilović played the main role for the work of seminar. After many years of preparation and delay, in 1905 the Great School has been transformed into a university that was "the highest self-administration body for higher vocational education and cultivation of science". The University sets eight regular professors who choose other teachers of the university. Mihailo Petrović was among these eight of professors, and Bogdan Gavrilović was immediately elected professor of the Technical Faculty. Thus

Petrović and Gavrilović got a leading role in the organization of scientific work and teaching at the newly founded university.

Further contribution to the development of teaching was the teachers' autonomy in the selection of programs and teaching without administrative intervention. In his St. Sava sermon *On enlightened idealism and its fostering with higher tuition*, which was held in January, 1901 in Captain Misa's building, in front of colleagues and the cultural elite of Belgrade, Bogdan Gavrilović advocated exactly such views. He defended liberal and independent spirit of the university by opposing Napoleon's absolutist and politicized opinion: "To him, the Napoleon, the main thing was to suppress free word at the university. Therefore, he particularly pointed his arrows at the spiritual sciences". In the same sermon, Gavrilović indicated that the holder of the development of science should be the Great School (which at that time did not become a university yet), and demanded it to strengthen its foundation and grow into a university: "... while at the University, particularly Faculty of Philosophy, which is the soul of his soul, the cult of pure science is the first and the last task. For these reasons the Great School, though we share so many beautiful memories from it, should broad and strengthen its foundations on which it will, with sight of the independent scientific research, morally and basically change our society and wind up our whole life with a new spirit – the spirit of enlightened idealism. Yes, we need a science research center, a university, a large school with the new direction and a new spirit, the school with an honor to, on a fresh and warm spring of science and truth, rejuvenate the intellectual, moral and political life of our people". Such public appearances and advocating of Bogdan Gavrilović have undoubtedly contributed to the development of science at the Great School and its turn into the university.

Gavrilović was at this time in his forties and at the top of his scientific work. He already had behind him two very valued books and a number of published high-quality scientific papers. That did not remain unnoticed and Gavrilović was elected a corresponding member of Academy of Sciences in 1901 and a regular member in 1905. Next year (1906) he was elected a corresponding member of the Yugoslav Academy of Sciences and Arts in Zagreb. At that time, he did not only publish works in the Academy's "Glas", but together with M. Petrović (and D. Nešić up to the 1904) proposed and reviewed papers for the journal.

By the 1909, the lectures in theoretical mathematics at the Faculty of Philosophy were held by Mihailo Petrović and Bogdan Gavrilović. That year, on their invitation proposal, Milutin Milanković came from Vienna to the Belgrade University to lecture an extensive subject of applied mathematics that was actually the course of mechanics, see [3] and [4]. In that period, Gavrilović gets high administrative functions of the University. Thus, in 1909 and the following year he had the function of the Dean of Technical Faculty. Furthermore, in the same 1910 he was appointed rector of Belgrade University and performed that high duty until the autumn of 1913. It is interesting to see which subjects Bogdan Gavrilović taught at that time (1906-1914): the theory of elliptical functions and their applications in algebra, geometry and mechanics, the theory of special determinants, and foundations of modern analytic geometry. In the meantime, the science at Belgrade University advanced so much that the first doctorate of mathematical sciences had been defended there. The doctorate in the field of differential equations defended Mladen Berić (1885-1935) in 1912, who was the suplent of the First Belgrade Gymnasium and Assistant of the Professor M. Petrović. The following year, Sima Marković defended a doctorate on a subject of Riccati's differential equations, also tutored by M. Petrović. Unfortunately the work of the Belgrade University was often interrupted because of the wars. School year 1912/13, the University did not work because of the Balkan wars. During the 1913/14 university was

reopened, but the First World War abruptly interrupted the development of the University. Students and professors went to war. In August 1914, just as the war started, in the bombing one part of Captain Miša's building has been demolished. The enemy looted the abandoned and demolished building.

Immediately after the war, in 1919, although in difficult circumstances, the University began to work. Education and university regulations were adapted to the needs of reconstruction and improvement of the country, as several generations of students and professionals were killed, and the University has been decimated in recent wars. In the early twenties, for a short time the university experienced rapid development: the number of teachers increased, and from Russia, among other professors came mathematicians Nikola Saltikov and Anton Bilimovič.

In 1921 Bogdan Gavrilović got a second term for rector of Belgrade University. In this position he remained three years, until the fall of 1924. Gavrilović at that time was completely turned to the general issues of culture, politics and history, as well to the place of science in other complex civilization achievements. His rector sermon "On the living forces of national unity" printed in 1922 in the journal "Serbian literary gazette" contains such notion. Many issues were discussed in this sermon: the war and peace and the reasons for starting the war just past, about democracy, about nationalism and national consciousness, of life with Croats and Slovenes in just created common state. The sermon is full of facts showing Gavrilović's encyclopedic breadth and universal spirit. Here's what he said, for example, on Vuk's struggle for national language:

"We usually say that in Vuk's struggle for national language the affirmation of folk consciousness was clearly outlined, but I think it had much broader and much deeper significance. National consciousness of our people who lived in Serbia, it is a specific consciousness, and it is different in its contents from national consciousness, for example, that part of the Serbian people who lived outside Serbia, and the biggest difference between them is that the national consciousness of the people in Serbia by its contents and characteristics always suited to the consciousness of the country, while in other sides of our people national consciousness previously was in complete contradiction with the consciousness of the country. And the true significance of Vuk's fight is that the victory of Vuk's principles made immediate and from the very start development of the whole public, civil and cultural life, or rather, must began to develop on the national basis."

And at the end of his sermon, Gavrilović seems to have a vision of some distant events in the future, which he described in the biblical words:

"Imagine - in a very bright point of your soul – a people which in the 5th century left their homes and - not knowing where to stop on its journey – came down from the Carpathian peaks into the Pannonian plateau, and further from there, into their nowadays countries; imagine the force of history split the people, on the crossing point between east and west; imagine how much, from that split, misery, suffering and devastation of all kind, how much trouble, cry, how much not deserved humiliation and insults of its pride, how much demonic dams on crystal clear streams that carried its spirit and its thought; - think in a word how overgrown in thorns and blood splattered was the path through which, in a series of long centuries, the life of the people shall pass; and look how through the mist that rose above the still hot and unsoaked blood of his – the only porphyry of his comfort, his hope, honor and dignity – is seen in the clear light the magnificent parade of the first chosen ones approaches with lighted candles and silent prayers to the eternity and before it on the altars of Rights, Justice and Freedom, creates national unity… There, in your vision stands the image of our history, which left to current generation in heritage the National unity. It is still there, among us,

but the light – the inner light – that resurrects the soul from darkness, is not among us, and peace – total peace – has not conquered our hearts yet."

In the mid-twenties a new generation of mathematicians matured: Tadija Pejović, Radivoje Kašanin, Jovan Karamata and Miloš Radojcic. Here are all the mathematicians who were at the Belgrade University in 1926. In the Faculty of Philosophy at the Department of theoretical mathematics were: full professors M. Petrović and N. Saltikov, assistant professor T. Pejović and assistants J. Karamata and M. Radojčić. On the department of Applied Mathematics: full professors M. Milanković and A. Bilimovič, Associate professors V. Mišković and assistant professor V. Žardecki. At the Technical University in the Department of Mathematics: Full Professors B. Gavrilović and P. Zajončkovski, assistant professor R. Kašanin; on the department of Applied Mathematics: I. Arnovljević and J. Hlitčijev. All teachers and assistants of the theoretical and applied mathematics from the University form the Club of Mathematicians of the Belgrade University. This seminar was actually a mathematical school of the University of Belgrade. It can be said that this was the golden age of Serbian mathematics. The club did not have any special rules, except that the meetings were held once a month and on that occasion the papers and scientific debates of club members were presented.

In 1926, Gavrilović reads his access academic sermon. On that occasion, Gavrilović was declared a regular member of the Academy. President of the Academy, John Cvijić gave the following explanation:

"Mr. Gavrilović is known worker in a large field of mathematics, and he worked independently on matters of geometry, algebra and theory of formal functions. I am particularly pleased to note that Mr. Gavrilović in the work "Analytical Geometry" led through the theory of finite-sectional the principle of correlation and thus continued what a German mathematician Hesse made in the theory of point, straight line and circle. Further, Mr. Gavrilović's works of considerable interest on different cultural issues, among which the problem of provisions of the values of civilization stands aside, as one of the problems of natural philosophy. On the basis of Article 14 of the Basic Law of the Academy I proclaim Mr. Gavrilović a regular member of the Serbian Royal Academy with my warmest congratulations."

Members of the Belgrade mathematical seminar printed their works mainly in the Academy's journal "Glas" and "Radovi" of the Yugoslav Academy of Science. Unfortunately, the rule of the Academy was that the papers must be published in the Serbian language, and so these works remained unknown to the wider world scientific public. Hence the need for a new magazine in which the works would be published in world languages was major. Thanks to the endowment of Luke Celovića – Trebinjac in 1932 a new journal "Publications Mathématiques de l'Université de Belgrade" was launched in which works were published in Russian, English, French and German. Until the Second World War seven volumes came out. So the scientific publications of Belgrade mathematicians became known to general mathematical public. In Publications, along with the Belgrade University mathematicians, world famous mathematicians published their works: E. Cartan, W. Sierpinski, P. Montell, J. Plemelj, H. Lemke, Dj. Kurepa, P. Erdös.

In 1929 Gavrilović retired but still teaches Analytical Geometry at the Technical University until April 1941. He was elected President of the Academy in 1931 and in this prominent duty remained until 1937. This late period of his life was marked by numerous social activities, ceremonial speeches and celebrations. Thus, he held speeches and wrote memorial articles dedicated to our and foreign great scientists: Goethe, Vuk Karadžić, Nikola Tesla, Mihailo Pupin, Dimitrije Nešić, Jovan Žujović, Paul Penleveu and others.

Bogdan Gavrilović was honored for his scientific and social work by the many scientific institutions and societies. As we said already, he was a regular member of the Serbian Academy of Sciences, the corresponding member of the Yugoslav Academy of Sciences and Arts, and member of society Circolo Matematica di Palermo, PhD. Hon. causa University of Athens, a member of Nikola Tesla Society, and from the 1939, when the Institute of Nikola Tesla was founded, the director of the institute.

Aside from science, Gavrilović had another love. On his rural property in Grocka he filed an orchard in which he grown peaches and other fruits. His contemporaries say that in this he followed and used the achievements of science, read the publications from this area and applied for that time modern agricultural techniques. According to Milutin Milanković, the fruit from his orchard was the best in the country.

World War II came, where in the calm of his life Gavrilović sees the ashes of books from the library which he founded together with M. Petrović fifty years ago. The war was gone, but his name, as one of the founders and members of the Scientific Council, still can be seen in the Chronicles of Mathematical Institute of SANU from 1947. The first session of Council of Mathematics Institute was held on June 22, 1946. On that occasion Bogdan Gavrilović, in his 83rd year was elected member of the Committee for abroad Relations.

Professor Bogdan Gavrilović died on August 6, 1947 in his native town, Novi Sad, and was buried in Belgrade. In the hall of the Mathematical Institute of SANU is bust of Bogdan Gavrilović.

3. Scientific work of Bogdan Gavrilović

With his scientific work Bogdan Gavrilović belongs to a specific time in more views. In the late 19th century, mathematics has grown into a high building with a large number of floors. New mathematical disciplines have been created, and for some of them might be said that they had already been completed. Specialization began to flourish on behalf of the scientific productivity. Bogdan Gavrilović, as a student who went to good European schools, gained an excellent mathematical education. Given the mentioned specialization, Gavrilović equally well knew and worked in several mathematical areas: algebra, analytic geometry and the theory of functions of complex variables. On the other hand, when he came to Belgrade there were a few pure mathematicians who were engaged in scientific work: Ljubomir Klerić, Petar Živković, Dimitrije Nešić, and Dimitrije Danić and before the end of the nineteenth century, Mihailo Petrović. At that time most of the population was illiterate, and the state through grants and sending abroad students for education, developed and supported more the practical sciences: engineering, construction, mining and legal science than fundamental science. The goal was to strengthen the young state economically and militarily. All of these facts we must bear in mind when evaluating scientific work of Bogdan Gavrilović, who, despite these circumstances, in the scientific work meet the standards of most developed European countries. He followed and linked to the works of the most famous mathematicians – contemporaries, while the themes he was interested in were new. And much later, when he has ceased to publish scientific papers, Gavrilović retained interest in contemporary research. And not only that, Gavrilović was also familiar with them, in which we can be persuaded from his 1926 academic sermon.

We should mention two more characteristics of Gavrilović's opus. All of his works, except the thesis, were written in the Serbian language and thus automatically inaccessible to wider scientific public. Gavrilović knew foreign languages very well, but according to the already mentioned Academy rule, the works published in its media had

to be written in Serbian language. Other characteristic refers to the length of the period in which he has published works. All of his scientific papers, with one exception, he published in a relatively short period 1900-1907. It can be understood why he did not publish before 1900, simply because he has been busy writing books from the extensive analytical geometry and linear algebra. But it is not quite clear why he stopped writing papers in 1907. In our publicist there are some interpretations, but the most acceptable explanation probably was that Gavrilović, as a man of wide interests and the universal spirit, in the second part of his life, has been more interested in topics from philosophy, history, education, and language. We should not forget the fact that at the same time he had dealt with high administrative and scientific functions which certainly distracted him to focus more on science.

3.1. Algebraic research. In his study of algebra, Bogdan Gavrilović was almost equally interested in three areas: combinatorics, number theory and linear algebra. In this research Gavrilović was, as he himself said several times, under the influence of English algebraists Arthur Cayley and and James Joseph Sylvester. They were the leading mathematicians in England in the second half of last century, and are important because their collaboration led to the creation of the theory of algebraic invariants. Their works have also drawn attention in Germany, so several German mathematician (Otto Hesse, Siegfried Heinrich Aronhold, Alfred Clebsch [Clebsch was the founder of the famous journal *Mathematische Annalen*] and Paul Gordan) continued this research with applications in analytical geometry. Gavrilović was familiar with the work of these mathematicians which can be concluded on the basis of numerous citations and references in his papers and books. Here are some detailed reviews of a few selected Gavrilovis's works in this field.

Very extensive article (53 pages), *"О тежинама алгебарских склопова"* (On the weight of algebraic sets) belongs to combinatory algebraic number theory, specifically to the partition theory. Let's recall that the main task of this theory was to determine the number of representation ("partitions") of the natural numbers as the sum of natural numbers, with addends or the number of summands that can satisfy a pre-given condition. A large number of famous mathematicians dealt with this, today popular field: Euler, Gauss, Cayley, Ramanujan, Erdösh. Gavrilović in his work determined the number of partitions of the natural number n on two, three and four summands (For example, the number of breaking the natural numbers n on two summands is $[n/2]$, the integer part of $n/2$). The method he used was elementary and reduced to very complicated numbering of certain subsets of natural numbers. Gavrilović did not seem to know the works in this area because there were no citations, while Cayley only two years before Gavrilović published paper (A. Cayley, *Researches in the partition of numbers*, (1898) Collected Math. Papers 2, see also: The theory of partitions, GE Andrews, 1976, Addison-Wesley Publ. Co.) in which the method for relatively easy solving of problems which Gavrilović set in his work was described. At the beginning of his writings Gavrilović mentioned that he came to this problem while determining the analytical expressions of some functions, but unfortunately does not give details so that we can make a comparison with the Cayley's analytical method of generating functions. It is interesting that Gavrilović expected that this theory would find its application in chemistry, "it will probably be applied in chemistry to calculate the number of organic compounds of a certain type" - said Gavrilović. The biggest value of this work is that Gavrilović seemed to independently recognize and solved the particular cases of a fundamental problem of combinatorial number theory. For the reader who will study this Gavrilović's article, let's say that the Serbian word "склоп"

(frame) in his work has the meaning of the modern term of "скуп" (set), while the *grade of frames* indicates the number of elements of that set.

The paper *On pfaffians,* although from the linear algebra, is essentially the work of combinatory character. Namely, Cayley proved ("Sur les déterminants gauches", Journal f. d. Reine und angew. Mathem. t. XXXVIII) that skew-symmetric determinant (those are determinants $|a_{ij}|$ where $a_{ij} = -a_{ij}$) of an even degree was a square of a polynomial. The polynomial is related to the famous Pfaff's problem of geometrical structure of solutions of differential equations, known as Pfaff's equations and according to Cayley, pfaffians. If $M = ||a_{ij}||$ is a skew-symmetric matrix of even order and Δ pfaffian matrix of M, then Δ is a linear function of the first kind of matrix M. Proceeding from these facts, Gavrilović gives essentially recursive definition of pfaffian through one set of permutations, which he describes in details. The definition with some modernized notation looks like this:

$$\Delta = \sum \varepsilon(s)\, x_{i_1 j_1} \dots x_{i_n j_n}$$

where $s = (i_1 j_1, \dots, i_n j_n)$ is a permutation of the set of partitions of {1, 2, ... ,2n} into two-element subsets $\{i_\alpha, j_\alpha\}$, where $i_\alpha < j_\alpha$, $\alpha = 1, \dots, n$, and $\varepsilon(s)$ is a sign of permutation s. It is not difficult to see that there are $(2n-1)!!$ such partitions, with two partitions s, s' are considered the same if

$$\{\{i_\alpha, j_\alpha\} \mid 1 \le \alpha \le n\},\ \{\{i_\beta, j_\beta\} \mid 1 \le \beta \le n\},$$

give the same partition of the set (1, 2, ... , 2n).

Gavrilović also discuss some algebraic properties of pfaffians analogous to the properties of determinants. For example, he fully discuss what happens if the signs of pfaffian elements change on odd or even diagonals. Namely, he shows that the sign of pfaffian changes depends only on the order of the pfaffian. It is interesting that the modern definition of pfaffian is very close to that description of pfaffian Gavrilović discussed in his paper (N. Bourbaki, Algebre).

In the paper *A new contribution to the theory of numbers* Gavrilović proves several theorems on sums of squares of natural numbers of which the most interesting reads: "Product of two sums with 9 squares can be expressed as a sum of 16 squares". With this he generalized several theorems, an old Euler's result, and one Langrange's formula, but also the results of several of his contemporaries (Genocchi, Arnoux, Antomari). In his quite elegant proofs he used elementary properties of complex numbers and the theory of determinants.

Gavrilović has also two papers on cubic determinants (these are "determinants" whose elements are indexed with three indices). In one paper he found an analogue of Sarus's rule for the development of cubic determinants of the third order. In the second paper he discusses what happens to the value of cubic determinant if the signs of the elements that lie in a horizontal, vertical or diagonal plane change. These are not great themes, but it should be mentioned that besides Gavrilović, with the problems of cubic determinants only two or three mathematicians in the world dealt with[1].

3.2. Research in analytical geometry. Gavrilović wrote three works from the analytical geometry. The first two came at the beginning of the century, and third *On maps of a coupled points of transfinite set of congruent projective sequences of points*

[1] For example, see *On Three Dimensional Determinants*, E. R. Hedrick, Annals of Mathematics, Second Series, 1:1/4 (1899 - 1900), 49–67; https://www.jstor.org/stable/pdf/1967268.pdf

was published in 1945 (with German translation). Gavrilović seem to have finished this third work much earlier, because in his academic sermon he looks at the main results contained in the paper "Looking for mappings of transfinite 'natural' sets I found that graphs of these mappings turned out to be hyperboles, but at the same time I proved with synthetic and analytical way that the families of these hyperboles cannot form a hyper-space of eight dimensions". This work is interesting because it tries to connect some geometric properties of figures with their set properties in terms of Cantor set theory. In the first two studies Gavrilović studies but some geometric properties of a family of conic sections (curves of the second order) as well as the features of some geometric transformation, especially projective, and the images of the second order curves obtained by applying these transformations. In these works it can be seen that Gavrilović had excellent knowledge of current researches in this area, which is not surprising because he has written a textbook from analytic geometry of 900 pages.

3.3. Researches in the field of theory of functions. In the field of theory of functions of complex variables, in addition to the thesis, Gavrilović wrote five papers. From today's viewpoint it is difficult to say whether his work in this area had important new contributions, but its results can never be considered trivial. It could be considered that these shifts were small, but not negligible.

Some subjects that Gavrilović wrote about are in focus today. For example, in the work *On analytical expressions of some functions* Gavrilović generalizes an Hadamar's result. Namely, the Hadamard express coefficients of analytic functions $1/f(z)$ with the determinants of function coefficients of $f(z)$, while Gavrilović generalizes the result on the function $g(z) / f(z)$. Gavrilović's proof is nice and not a direct transfer of Hadamar's argument. In the work *On some trigonometric identities*, Gavrilović proves rather complex trigonometric identities using the method of residue arithmetic. Let's mention there is no direct way to prove these identities. In another work he varies the theme by proving trigonometric identities using the properties of symmetric functions of roots of cubic equations. In the work *On the residue of one-branch functions* Gavrilović is linked to the Weierstrass results in relation to the rest of logarithmic derivative and essential singularities of analytic functions. He introduced weight in logarithmic derivatives and for that generalization got similar results as the Weierstrass. In the discussion *On analytical representation of one-branch functions in a point domain of infinity* Gavrilović generalizes some Ermit's results relating to the residues of functions from one class of rational functions. This work is well written with elegant proofs and contains result which are still of interest today. Let's mention that this paper of Gavrilović Vivanti quotes in his textbook.

3.4. Academic sermon. In his access academic sermon "The problem of space, hyper-space and continuum", which was held on March 7, 1926, Gavrilović presented his view of mathematics and the problem of establishing some basic mathematical concepts. This presentation goes in several directions. In the first part Gavrilović analyzes the concept of space in geometry and in Euclid geometry, Ryman geometry and geometry of Lobachevski. He praised the intellectual bravery of Lobachevski for rejecting the 5th postulate (on the parallel straight lines) from Euclid's axiom system and thus introducing geometry with the same exactness and non-contradiction like Euclid's geometry. And here's what he thinks about Ryman geometry as a mathematical model of physical space:

"Although Riemannian geometry seems paradoxical, the new theories of modern physics develop in its system thereof. But even without that, the geometry, as the ideal geometrical system, represents the model of geometry of the finite World."

Gavrilović also presents the problem of space as seen in classical (Newtonian) mechanics, but also in the Einstein theory of relativity. He is well acquainted with contemporary works from the foundation of geometry (Hilbert, Poincare, Minkowski and others). He is familiar with the example of finite geometry – when mentions "ramification of system of only 7 points which collects in one system all the points of space", probably thinks, although not explicitly mentions, of the finite model of Gino Fano from 1892, so called the first group of axioms of projective geometry (incidence axioms) which has a total of 15 planes each of which is incidental with 7 points and 7 straight lines.

In the second part of the sermon Gavrilović deals mainly with the Cantor set theory as a universal framework for the establishment of a real continuum, analysis and geometry. With regard to that he cites numerous problems and examples from cardinal arithmetic, such as the continuum hypothesis (that each infinite subset of the real continuum is equipotent to the set of natural numbers or to the set of real numbers), or an example of Peano's curves with which the edges of the square can be continuously transferred to the whole square.

Another theme permeates the sermon. That is the problem of infinity and how that concept is founded in mathematics. Gavrilović points out that various aspects of the intuitive notion of infinity in various ways reflect in different areas of mathematics: one way in geometry, the other in the analysis and the third way in Cantor set theory. Thus, in one place he said that the "infinite calculus is the deepest and most beautiful lyrical song about infinity", although he believes that the deeper essence of the infinity can be reached only by establishing the basic concepts of the analysis, such as continuity, for example. However, according to him the right framework for the foundation of the concept of actual infinity in mathematics is the Cantor set theory. Namely, he paraphrases Hilbert, considering that this theory is "a paradise from which the problem of the infinite cannot be repressed, but it will in the future, for a long time yet, be one of the most prolific areas of mathematical speculation". Gavrilović is quite right about that. In the other place Gavrilović wonders if there is a continuum and the infinity in the reality. It is believed that the answer to that question is negative. When it comes to infinitely small sizes, the confirmation of his opinion he found in the indivisibility of elementary particles, i.e. the existence of the smallest amount of energy (Planck's quants). So in the real world there are no infinitely small quantities, so there is no homogeneous continuum. For the fact that there in the physical world there is no infinitely large size, Gavrilović finds evidence in the finite Einstein cosmological models. Further, Gavrilović said, "The Mathematics can not say this: space is infinite, it cannot say space is finite either". In fact, he believes that the most important notion in mathematics is *derivation* or *proof*, and thus received truth did not in any case prejudice the nature of space, regardless of what starting assumptions were. In this sense Gavrilović's position is close to Hilbert's formalist terms, according to which the concept of infinity in mathematics is useful but the fiction that can be eliminated. This attitude is seen in another place in the sermon when he says: "For on the axioms we can think that we want, we can say for them, that they are conventions or the statements *a priori*, we can accept some of them, and some reject, but when some of them have already accepted, then in one and in the other lap it, which is to be developed from them, must be logically true".

Let's mention that in this document, Gavrilović tried, as in some earlier works, to connect, not quite successfully we might say, some geometric properties of space with cardinal arithmetic Cantor set theory. The truth is that someone here can see hints of a discipline that will appear thirty years later - the theory of models, where these issues will be discussed on other grounds.

It is interesting that Gavrilović's sermon had not a particular echo in the then Belgrade circle of mathematicians, although it did not display only the author's views, but the actual facts and the views of some of the greatest modern authorities in this area (Russell, Hilbert, Poincare, Cartan, Eddington and others). One reason probably is that most of Belgrade mathematicians, if not all others, were sent to study and solve specific mathematical problems, primarily from the analysis. Simply, in Belgrade at the time was little interest for the geometry and foundation of mathematics. Since it's been 70 years since the occurrence, Gavrilović's sermon seems still fresh and funded and may be timely for all those interested in issues from the foundation of mathematics. Also it shows that Gavrilović was very familiar with the works on this subject of the authors of his time, not only in mathematics but in physics and philosophy.

4. Gavrilović's books

Gavrilović has published two books. The first book, *Analytical geometry* was published in 1896. The second book, *The theory of determinant*s was published in 1899. Both books have many common characteristics. The first characteristic refers to the terminology. As one of the first writers in Serbia in this field (the first university textbooks in mathematics of the general type wrote Emilijan Josimovic in the sixties of XIX century, while later Dimitrije Nešić wrote slightly specialized textbooks) Gavrilović had to have certain problems of terminology. This raised the question of whether to introduce new Serbian words or accept foreign "serbified" terms. These dilemmas may be best described by the words of Emilijan Josimovic, one of the pioneers of our science, which at one place says: "Finally, in the observation of the language I honestly admit that I am still very weak, but also quite naturally, because it was such an opportunity, that I, as many other Serbs except *Часловац* (a form of prayer book) and Psalters, had to learn everything in a foreign language". Gavrilović has successfully solved many problems of terminology. Here are some examples of words that he used and which would look old or unusual in Serbian today: *еквације* (equations), *писмено* (variable), *казаљка* (index, it seems that the term "казаљка" was introduced by Dimitrije Nešić), *прецрт* (set image, mapping), *рој* (family), *склоп* (set). Another characteristic of these books are numerous historical notes that Gavrilović used to explain the origin of important mathematical terms. Also, in many cases where we can find the names of the authors of the theorems and the tasks that are now forgotten or considered mathematical folklore. Thus, in *Analytic geometry* we find among many others one Bošković's mathematical problem, while in *The theory of determinants* we know that the word determinant comes from the Gauss. Books abound in these details, which still give them a special value, especially bearing in mind that contemporary authors rarely give explanations of this kind.

Gavrilović wrote these books with great pretensions, as he himself says in the preface to *The theories of determinants*: "I wanted to bring up the whole basis of the theory together with the theory of specially important determinants, but I was especially careful, that those who would like to enter deeper in the modern analytical geometry, to get the best possible basis".

The books have been written with a beautiful and consistent style. The proofs are strict and from the standpoint of modern mathematics correct. However, the writer saw before him a regular student, and the theorems are often illustrated by examples, and proofs, where the idea could be maintain, instead of the general case ("for arbitrary n") carried out in special cases (for n = 2 or n = 3). So the books are easy to read and can serve as a model of good textbooks. According to the testimony of Professor Zlatko

Mamuzić, *The theory of determinants* has been used as a textbook until the Second World War.

4.1. Analytical geometry. Complete name of this book is "Analytical geometry of straight, round and conic section". Gavrilović devoted the book to Serbian benefactor Sava Tekelija, of whose endowment he was once a scholar. At the very beginning, Gavrilović points out that a book was written for those who want to learn about the method punctual and tangential geometry. Actually, the biggest and main part of the book refers to the projective geometry of the plane and the study of conics (conic or conical intersection). Even in times of ancient Greece, and later, especially in the 19th century, the conical sections were systematically studied and a large number of results on these lines were obtained. This is understandable, given how often these forms occur in mathematics, mechanics, physics, astronomy, technology, architecture and other areas. Let's recall that planets and comets, for example, travel on the trajectory that can be described (in the first approximation) as conic. Gavrilović's book is an encyclopedia not only of the results of these curves, but also the methods by which those results were obtained. Given the scope of work, the book has over 900 pages, this time we cannot go into its more detailed analysis. However, let's say the following.

Work is divided into six books, and only for the first book and first half of the second book can be said that for the most part belong to elementary analytic geometry, although here are discussed less elementary topics, such as foundations of the transcendent theory of algebraic curves. Thus, for example, in the rectangular coordinate system, but also in polar coordinates of the equations and main properties of algebraic curves of the second order are given: circle, ellipse, hyperbola and parabola; third order: cissoids and strophoid; fourth order: conchoids real and conchoids circle; transcendent curves: Archimedes' spiral. In the section point and tangential coordinates, Gavrilović introduces projective coordinates first of the straight line and the so-called tangential coordinates of the line. Thus it gives a natural analytic and geometric motivation for this definition, it is ordered pair (u, v) of parameters u, v in a normal equation $ux + vy + 1 = 0$ of the straight line. These coordinates are known under the name Plücker's coordinates of the straight line, according to Julius Plücker (1801-1868) who introduced them in 1829. The same coordinates are called the homogeneous or line coordinates of the straight lines (for example see Mileva Prvanović, *Projective geometry*, Belgrade, 1986), if we take triple (u, v, w) of the parameters u, v, w in general equation of the straight line $ux + vy + w = 0$. At the same Gavrilović then explains his sentence in the introduction, or what the punctual means, and what the tangential geometry (p. 196) is:

"Geometry in which the basic element of the images is point we shall call punctual geometry, and geometry in which the basic element of the images is straight line will be called tangential geometry."

In the next section, the so-called trilinear (or homogeneous) coordinates of point in the projective plane are introduced. From that place until the end of the book, that is some 700 pages, analytical method of projective coordinates in solving various geometrical problems and testing conics is used. In this, Gavrilović uses the principle of correlation, i.e. the principle of duality as a general principle for the doubling of geometrical theorems, as they were first formulated and applied by Poncelet, Chasles, Gergonne and Möbius in the first half of the 19th century. Here's what Gavrilović says about that (p. 208): "When the results we got with the help of tangential coordinates are compared with some results we obtained using parallel coordinates, it will be seen that between these results there is some certain reciprocity. By the reciprocity it can be seen that the geometric theorems can be easily transferred from punctual in tangential

geometry; in some theorems instead of words point, straight line, place, points where straight lines cut etc. the words straight line, point, envelope, straight lines connecting the dots, etc. These two theorems are called dual or correlations theorem, and a principle on which these theorems can be ingeminated, principle of duality, or the principle of correlation."

The book has 442 references, which contain hundreds of theorems, examples and tasks. The results from the classical works, memoirs significant, but lesser-known debates, of the mathematician are exhibited: Jean-Victor Poncelet, Joseph Diez Gergonne, August Ferdinad Möbius, Jakob Steiner, Arthur Cayley, Otto Hesse and others, in all over 100 names. Here are a few examples. Here we find an elegant and short analytical proof using trilinear coordinates that Euler's straight line of the triangle goes through the ortho-center, centroid and center of the circle of the triangle described. Then, there is two analytical solutions of Apolonie's problem: find a circle K which touches three circles, one belonging to Hesse. Dual Brianchon's and Pascal's theorem about the properties registered and described hexagon of conic section have been proved. The elements of Cayley's theory of invariants are exposed. This voluminous book contained, I do not know whether we can say all the knowledge of conic and geometry of projective plane, but certainly a huge part of that area by the time it was written. There are many reasons why every mathematician, especially a geometer and algebraists would be happy to have in his hands. There he can find many, maybe even the forgotten theorem and problems, and in many cases the name of the mathematician who proved the theorem, i.e. set or solve the problem. It gives an everlasting value of this book, especially bearing in mind that in modern textbooks most of these names lies in the darkness of anonymity. Gavrilović introduces new terms, giving a natural motivation, without bulky labels and with a lot of beautiful drawings. It may be the reason why the part of generality or accuracy is sometimes lost, but the reader can easily follow mathematical text and main mathematical ideas. The book is written in beautiful and picturesque language and we can easily say that it belongs to a beautiful mathematical literature.

4.2. The theory of determinants. This book represents the first serious work in linear algebra in the Serbian language. Indeed, Dimitrije Nešić published so early as 1883 the university textbook "Algebraic analysis". This is the first time in the Serbian mathematical literature that the part of the theory of determinants and some applications of this theory in algebra have been presented. Gavrilović included this book in a bibliography citing Nešić's name (as the only Serbian author). Given the time and circumstances when it appeared, Gavrilović's book seems surprisingly modern in presentation of linear algebra, introduction of terminology, review of connections between linear algebra and other areas of mathematics and historical comments. Gavrilović writes in the foreword: "…on that part this theory of determinants will hopefully take a respectable place in our poor scientific literature". A little further Gavrilović also says "there were no printing errors and in that this work can equate with the best English editions". Gavrilović's exposure of the theory of determinants is of combinatory character. At the very beginning the author presents an algebra of permutations, so that the determinant would be defined as the combinatorial function of its elements, as this concept is commonly introduced today using permutatuins:

$$\Delta = \sum \pm a_{1p_1} a_{2p_2} \dots a_{np_n}$$

At the time of occurrence of this book, it was known that the set of permutations of the domain forms a group. However, Gavrilović does not expose this part of the theory of permutations. On the other hand, he distinguishes the concept of matrix and the concept of determinant as a function defined on square matrixes. At that time, unlike the determinant, the concept of a matrix is a new concept. The book presents the standard theory of determinants, like Laplace's theorem, properties of minors and cofactors, then application on solving the systems of linear equations (Cramer's theorem, the theorem on existence of non-trivial solution of homogeneous system of linear equations). The special value of the book provides resources and application of the theory of determinants in other areas of mathematics. For example, in analytic geometry, (in determining the equation of the plane through three non-collinear points, in the second part of the book in solving of axis problem for curves of the second order) also in the algebra for determination of the resultants of algebraic equations. There are examples in the analysis as well: the property of jacobians is studied (the theorem on inverse function is being proved), hessians and wronskians (differential determinants) with the application on the theory of linear differential equations (in reduction of the order of equation if the particular integral is known). Here we can glimpse the elements of functional analysis, because in one place Gavrilović mentioned linear independence of functions and representation of this relation through wronskians.

In the book, the features of special determinants (matrixes) are exhibited with quite a detail: symmetric, ortho-symmetric, skew-symmetric, circulants, pfaffians, alternants and continuants (with application in the study of continued fractions). The last third of the book is devoted to the theory of linear transformations in Euclidean spaces. Although Gavrilović has no concept of vector space, he implicitly gives the matrix representation of linear operators of finally-dimensional vector spaces. Orthogonal operators he defined geometrically, as linear transformations that preserve the vector norm, and then gives algebraic equivalents, for example, that $A^{-1} = A^T$, where A is a matrix operator. Here we shall also find Binet-Cauchy theorem, then that the matrix of a product of linear transformation is equal to the product of their matrix, or in today's terms, the ring of endomorphisms of n-dimensional vector space is isomorphic to the matrix ring of order n (of the scalar field space).

The book also presents the theory of quadratic forms. The procedures for the reduction of quadratic forms to diagonal forms (Gauss' and over orthogonal transformation) are given. The Sylvester inertia theorem is also being proven.

The last, eighth chapter refers to the theory invariants and covariants. That was, at that time, very modern and actual theory that was mainly developed by English and German mathematicians. In Gavrilović's book we can find a description of invariants of square form and, implicitly, Gordan's theorem according to which to binary form belong the finite system invariants and covariants. Let's recall that Hilbert generalized that theorem on arbitrary algebraic forms ("The finite basis theorem") and that is considered the creation of algebraic geometry. Further, in the book it is proved that hessians and jakobians are covariants, and here we find elements of tensor algebra. It should be borne in mind that G. Ritchie formulated covariant and contra-variant laws no sooner than 1889. and the widespread adoption of the term "tensor" in physics and mathematics comes from Einstein's usage in general relativity around 1916 (Annalen der Physik t. 49, 1916).

This interesting book was very modern for its time, we can say at the level of Europe's best books of the time. Of course, as the book reflected the actual mathematics of its time, some now standard topics and areas are only implied in the book or do not exist (such as the notion of vector space and matrix rank). However, the writer of this preview read this book with pleasure.

5. Epilogue

If we give a summary on the life path, and particularly on the scientific and cultural work of such persons of reputation as it was Bogdan Gavrilović, we can fall into the trap of wrong and sketchy grade, or at least those that are not acceptable for everyone. However, that risk of a mistake we must accept, but will also introduce evaluation and opinions of some of our distinguished mathematicians. When talking about the time in which Bogdan Gavrilović began its work in science, and the age of his life when he was able to give its best to science, we must bear in mind the opportunities that ruled Serbia and Europe at the end of the last century. Serbian people lived abroad mainly, while in Serbia itself, science and culture were still being born. Material resources that Serbian state could give for science development were very modest. On the other hand, in the nineteenth century, especially in the second half of the century, the foundations of modern mathematics were set. New mathematical theories with many applications in technology and physics are being born while older ones are strictly logically based. It is impossible at this place in a few words to prove - it is an enormous issue for itself. But let's at least mention two examples: the Weierstrass with its ε-δ calculus formally based analysis, while Cantor built set theory and with it gave frame and universal language of modern mathematics.

Bogdan Gavrilović, a direct student of Weierstrass, starts his career armed with this scientific knowledge and understanding of European science and culture. Although very young, as a mathematician he was already established. He was inclined towards geometry and knew very well the works of English and German algebraists, while in the works from the theory of functions; he discussed the most pressing questions. On that occasion, Miroslav Pavlovic, our distinguished expert in complex analysis says that according to the knowledge of the theory of functions that Gavrilović had, then and now he could be a professor of the subject in any European university, and as a scientist he is absolutely a man who had a mathematical mind and talent. Already in the 35th year Gavrilović had behind him two books, one from analytic geometry, and the other from linear algebra, the total volume of 1250 pages. This is to repay the students and the university as a teacher. According to academic Radivoje Kašanin "both, especially the last one committed honor to every nation, and many people at that time, bigger and happier than us, then, such acts did not have".

Gavrilović wrote for a short time twenty high quality works. However, Gavrilović's works, perhaps undeservedly, have not left a big mark in Belgrade mathematical environment, at the international level even less, because they were all published in Serbian language. Gavrilović did not have doctoral students, nor did he leave direct students behind. We can say that his work in science remained in the shadow of another great Serbian mathematician, his friend and colleague, Mihailo Petrović, with whom he was practically alone in Belgrade mathematical scene for 15 years from the 1894 (when Petrović came to Belgrade for a professor) to 1909 (when Milutin Milanković comes to the University of Belgrade). One view of this situation, give academician Miodrag Tomic, that the scientific work of Gavrilović represented a transition from Dimitrije Nešić to Mihailo Petrović. Here we must repeat that themes of Gavrilović's works were contemporary for its time, and that some of his research results, today, almost a hundred years from the time of their occurrence may be of interest.

But Gavrilović was not only simple and retiring university professor of mathematics. He was also an excellent organizer of scientific work who very much contributed to turning the High School to Belgrade University, and to the creation of

library of Mathematics seminar, which will be one of the bases of the scientific work of Belgrade mathematicians. Academic Gavrilović was an eminent educational and cultural worker who had thought and opinion on many important issues in the life of his people, politics, history and philosophy. As a man of universal spirit, he did not observe his science in isolation, but in light of other sciences and cultural heritage. His speeches testify about that, numerous articles and sermons in various important occasions from Belgrade's social and scientific life. As a member of the narrowest Belgrade's intellectual circle, he has contributed to creating a special atmosphere thanks to which Belgrade as the provincial town he once was, became one of the centers of scientific work. Together with Mihailo Petrović and Milutin Milanković he introduced modern mathematics in Serbia. In the words of Radivoje Kašanin, his assistant and successor at the Department of Mathematics Technical School, who described the atmosphere of the place like this: "In addition to the high education and original research papers, all three were honored with something that I appreciate most, what I consider the human value of the highest rank: the love for the young generations, understanding young people, selflessness and sincere assistance to young talented people in their progress. They were able be happy and to enjoy when young people are rising. *I had such luck to develop and work beside them, the great authorities of science and morality. To be proud of their friendship. I do not believe that ever was such a climate as created by Gavrilović, Petrović and Milanković*".

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